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ABSTRACT

Hypothetical data sets are used to demonstrate how canonical correlation methods subsume other commonly utilized parametric methods. Analysis of variance, analysis of covariance, multiple analysis of variance, and multiple analysis of covariance are heavily used by educational researchers. It is concluded that researchers would do well to consider using general linear model (GLM) techniques as opposed to analysis of variance and its analogs. GLM and analysis of variance are equivalent when the ways in an analysis of variance design each have exactly two levels. In other cases, the GLM analytic approach yields more specific information regarding effects, and greater power against Type II error, because the comparisons are planned a priori rather than post hoc. Multivariate canonical correlation methods are superior to univariate techniques in providing important information on the interrelationships among variables. Furthermore, multiple univariate procedures tend to inflate the probability of a Type I error. (GDC)

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HEURISTICS FOR TEACHING

MULTIVARIATE GENERAL LINEAR MODEL TECHNIQUES

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ABSTRACT

Mathematical statisticians have long recognized that parametric significance testing procedures are special cases of canonical correlation analysis, but most research practitioners have been less aware of these relationships. The paper presents a series of actual analyses of a hypothetical data set in order to illustrate these relationships. Insight into the linkages among various techniques should facilitate better understanding of methods and more informed analytic practice in educational research.

Kerlinger (1973, p. 216) once noted that "the analysis of variance is not just a statistical method. It is an approach and a way of thinking." Much the same can be said about statistical techniques as a system. Cronbach (1957, p. 671) recognized this reality in his presidential address at the sixty-fifth annual meeting of the American Psychological Association:

> Two historic streams of method, thought, and affiliation... run through the last century of our science. One stream is experimental psychology; the other, correlational psychology... Psychology continues to this day to be limited by the dedication of its investigators to one or the other method of inquiry rather than to scientific psychology as a whole.

Behavioral scientists who prefer to think of themselves as experimenters historically have preferred to analyze data using analysis of variance (ANOVA) methods and their analogs (ANCOVA, MANOVA, and MANCOVA--collectively here labelled OVA methods). Edgington (1974, p. 25) studied seven prominent journals published by the American Psychological Association, and reported that in 1972, "71% of the articles using statistical inference used analysis of variance." Willson (1980, p. 6) studied the 1969 through 1978 volumes of the American Educational Research Journal and found that "ANOVA and analysis of covariance (ANCOVA) comprised over 34% of the total techniques and were included in 56% of the articles." More recently, Goodwin and Goodwin (1985) found that 37% of the articles in the 1979-1983 volumes of the American Educational Research Journal employed OVA techniques, while during the same period 61% of the articles in the Journal of Educational Psychology used OVA methods.

In some respects these patterns are disturbing, since OVA methods have been criticized for distorting research results (Cohen, 1968; Thompson, 1981). The pattern is also somewhat surprising, because increasing numbers of researchers have come to recognize that general linear model methods subsume all parametric significance tests. For example, Cohen (1968, p. 426) noted that ANOVA and ANCOVA are special cases of multiple regression analysis, and argued that in this realization "lie possibilities for more relevant and therefore more powerful exploitation of research data." Kerlinger and Pedhazur (1973) point out that general linear model implementations of OVA methods force planned comparisons. They also note that:

> The tests of significance for a priori, or planned, comparisons are more powerful than those for post hoc comparisons. In other words, it is possible for a specific comparison to be not significant when tested by post hoc methods but significant when tested by a priori methods. (p. 131)

Since Cohen's article several excellent texts (Edwards, 1979) on these methods have been presented, and the regression approach to OVA "has been extensively used" in recent research (Willson, 1982, p. 1).

However, canonical correlation analysis (Hotelling, 1935), and not regression analysis, is the most general case of the general linear model (Baggalley, 1981, p. 129). Fornell (1978, p. 168) notes that "multiple regression, MANOVA and ANOVA, and multiple discriminant analysis can all be shown to be special cases of canonical analysis. Principal components analysis is also in the inner family circle." Knapp (1978, p. 410) demonstrated this in some mathematical detail and concluded that "virtually all of the commonly encountered parametric tests of significance can be treated as special cases of canonical correlation analysis."

The purpose of this paper is to demonstrate by way of examples exactly how canonical methods subsume other commonly utilized parametric methods. Knowledge of these linkages is important--the knowledge provides researchers and students with real insight into the similarities and differences among methods. Most importantly, knowledge of the linkages provides a framework for better understanding the techniques both individually and collectively.

Table 1 presents a hypothetical data set (n = 12) used to make the presentation of examples concrete. Readers may wish to replicate the reported analyses using whatever statistical packages are preferred. Results are reported in some detail throughout the paper in order to facilitate such comparisons. Unless specifically noted otherwise, results are reported exactly as presented on the printouts generated using the Statistical Package for the Social Sciences (Nie, Hull, Jenkins, Steinbrenner & Bent, 1975). The X and Y variables in Table 1 represent hypothetical intervally scaled dependent variables. Variable A represents a hypothetical intervally scaled independent variable, while B represents a variable designating hypothetical assignment to experimental conditions.

INSERT TABLE 1 ABOUT HERE.

Variable A′ is a trichotomy derived by converting A to the nominal level of scale in the same fashion that many OVA researchers treat aptitude variables in aptitude-treatment interaction OVA designs. Variables A1 through A2B1 are "coding columns" that exactly restate the information originally presented in the variables from which the columns were derived (A1, A2 from A′; B1 from B; A1B1, A2B1 from A1, A2, and B1). The coding columns are perfectly uncorrelated or "orthogonal." This can be readily seen by noting that the mean (via the sum) of every coding column is zero and that the sum of the cross-products of any two columns is also zero. The raw score formula for the correlation coefficient presented in statistics texts indicates that these two conditions will delineate orthogonal variables. The A1B1 and A2B1 coding columns were derived by multiplying, respectively, the A1 column times the B1 column, and the A2 column times the B1 column. These two variables measure the interaction effects of the A and B ways on one or more dependent variables.

Table 2 demonstrates that canonical correlation analysis (CCA) subsumes the t-test, developed by Gossett under the pseudonym "Student" in 1908 (Cooley & Lohnes, 1971, p. 223). For the purposes of this analysis Y was declared the dependent variable while group membership delineated by B (either 0 or 1) was the independent variable. The t-test was performed in the usual fashion. The mean of group 0 (7.8; SD = 2.8) was not different enough from the mean of group 1 (5.2; SD = 4.1) for the difference to be considered statistically significant at most conventional alpha levels. The canonical analysis was performed by indicating to the computer that the CCA involved only one dependent variable (Y) and only one predictor variable (B). The CCA routine used a different test statistic than the t-test. However, it should be recognized that all of the various test statistics (Z, t, chi-square, F, etc.) are related, as explained by Glass and Stanley (1970, pp. 236-238). The calculated probabilities for the two sets of results are identical.

INSERT TABLE 2 ABOUT HERE.

Table 3 demonstrates that CCA subsumes the Pearson (1901) product-moment correlation coefficient. Thompson (1984b, pp. 14-16) provides substantially more detail on this linkage by presenting CCA in bivariate terms. Both analyses provide identical results when Y and A are related, though different test statistics were employed.

INSERT TABLE 3 ABOUT HERE.

Table 4 presents a conventional factorial ANOVA (Fisher, 1925) that considered Y as the dependent variable and A´ and B as the ways in the design. The implementation of ANOVA utilizing the CCA routine requires that four sets of analyses be conducted. The analyses are presented in Table 5. The first analysis predicts Y with the five coding columns (A1 through A2B1) developed to represent the three effects in the ANOVA design (A´ main effect, B main effect, and the two-way interaction). The remaining three analyses on a rotational basis exclude the coding columns associated with each of the three effects in the ANOVA design.

INSERT TABLES 4 AND 5 ABOUT HERE.

These analyses are conducted to obtain the Wilk´s Lambda values reported in the table. Lambda is analogous to a sum of squares in a conventional ANOVA. It is an estimate of an effect. Unlike an SOS, however, as Lambda gets smaller the effect is larger rather than smaller. Table 6 presents the mathematics for converting the Table 5 values into effect estimates that correspond to the three effects in the ANOVA results. It should be noted that these Lambda´s are inversely related to the SOS´s for the effects presented in the Table 4 ANOVA keyout. Table 7 uses the algorythm suggested by Rao (1951) for converting these values to F test statistics. Again, the results across the two analytic methods agree perfectly.

<u>INSERT TABLE 6 AND 7 ABOUT HERE.</u>

Table 8 demonstrates that CCA subsumes multiple regression analysis. The analysis involved the prediction of <u>Y</u> with <u>X</u>, <u>A</u>, and <u>B</u>. The results correspond though different test statistics are employed. Table 9 demonstrates that the function coefficients produced by the CCA are related to the regression beta weights, even though the two sets of coefficients at first pale appear to be different. The difference is that a variance adjustment utilizing either <u>Rc</u> or <u>R</u> is necessary to equate the coefficients across the two methods. Thompson and Borrello (in press) provide more detail on these relationships.

<u>INSERT TABLES 8 AND 9 ABOUT HERE.</u>

Table 10 demonstrates that CCA subsumes discriminant analysis. The result is not surprising. Tatsuoka (1953) demonstrated these relationships long ago, and the computer even labels the discriminant functions, "canonical discriminant functions." The relationships between the two sets of function coefficients are less obvious. Harris (1975, p. 143) explains the relationship:

> What meaning is to be attached to the coefficients by which Canona [CCA] tells us the group membership dummy variables are to be weighted? From the maximization criteria which define Canona, it follows that they give the contrast among the means which accounts for the greatest percentage of the between group variability in the discriminant function.

<u>INSERT TABLE 10 ABOUT HERE.</u>

Table 11 presents the canonical variate scores derived by applying the canonical function coefficients presented in Table 10 to the variables Y and X in their Z-score form. Table 11 also presents the discriminant function scores derived in an analogous manner. In addition to being perfectly correlated, the values become identical if the discriminant function scores are standardized through division by their standard deviation.

INSERT TABLE 11 ABOUT HERE.

Tables 12, 13 and 14 demonstrate that CCA subsumes MANOVA. The presentation is parallel to the demonstration involving ANOVA. However, the presentation is somewhat simplified since the MANOVA effects are already presented as Lambda's.

INSERT TABLES 12, 13, AND 14 ABOUT HERE.

Discussion

The considerations presented in this paper suggest that researchers would do well to consider use of general linear model (GLM) techniques as against their OVA analogs. The GLM techniques are exactly equivalent when the ways in an OVA design each have exactly two levels. Otherwise, the GLM analytic approach yields more specific information regarding effects and greater power against Type II error, thanks to the planned comparisons discussed previously.

With respect to the use of univariate techniques rather than CCA or its special multivariate cases, the multivariate methods are superior. The methods provide substantively important information regarding how all the variables interrelate. Futhermore, the use of multiple univariate procedures within a single study tends to inflate experimentwise Type I error probability. The actual experimentwise Type I error probability in such cases ranges between the nominal alpha used for each univariate analysis and 1 - [(1 - alpha) raised to to k power], where k is the number of dependent variables in the study (Morrow & Frankiewicz, 1979, p. 299).

Krus, Reynolds, and Krus (1976, p. 725) argue that, "Dormant for nearly half a century, Hotelling's (1935, 1936) canonical variate analysis has come of age. The principal reason behind its resurrection was its computerization and inclusion in major statistical packages." In an annotated bibliography, Thompson (1984a, p. 3) recently noted that:

> A noteworthy indication of increased interest in canonical methods involves the recent publication of several articles which explain canonical methods in conceptual or essentially non-mathematical terms. It is particularly noteworthy that these peices represent journals from such disparate disciplines.

Thus Wood and Erskine (1976) were able to review more than 30 applications of these methods. These developments may be happy ones since "some research problems almost demand canonical analysis" (Kerlinger, 1973, p. 652) and since "it is the simplest model that can begin to do justice to this difficult problem of scientific generalization" (Cooley & Lohnes, 1971, p. 176).

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Table 1

Hypothetical Data for Heuristic Demonstrations

Y	X	A	B	A´	A1	A2	B1	A1B1	A2B1
1	11	5	1	2	+1	-1	-1	-1	+1
2	5	3	1	1	-1	-1	-1	+1	+1
3	2	2	1	1	-1	-1	-1	+1	+1
4	8	8	0	2	+1	-1	+1	+1	-1
5	4	4	0	1	-1	-1	+1	-1	-1
6	12	10	1	3	0	+2	-1	0	-2
7	7	6	1	2	+1	-1	-1	-1	+1
8	1	1	0	1	-1	-1	+1	-1	-1
9	9	12	0	3	0	+2	+1	0	+2
10	3	7	0	2	+1	-1	+1	+1	-1
11	6	9	0	3	0	+2	+1	0	+2
12	10	11	1	3	0	+2	-1	0	-2

Table 2

CCA Subsumes t-tests

[Y by B(0,1)]

Canonical Analysis		t-test Analysis	
Squared Rc	.14918	Mean of Group 0	7.8333(2.787)
Rc	.38624	Mean of Group 1	5.1667(4.070)
Lambda	.85082		
chi-square	1.53482	t	1.32
df	1	df	10
p	.215	p	.215

Table 3

CCA Subsumes Pearson r

[Y with A)

Canonical Analysis		Pearson Correlation	
Squared Rc	.32085		
Rc	.56643	Pearson r	.5664
Lambda	.67915		
ch-square	3.67563		
df	1		
p	.055	p (one-tailed)	.055

Table 4

Factorial ANOVA

[Y by A´(1,3),B(0,1)]

Source	SOS	df	MS	Fcalc
A´	56.000	2	28.000	2.754
B	21.333	1	21.333	2.098
A´B	4.667	2	2.333	.230
Error	61.000	6	10.167	
Total	143.000	11	13.000	

Table 5

Canonical Analyses Using Four Models

Model	Predictors of Y	Lambda
1	A1,A2,B1,A1B1,A2B1	.42657
2	B1,A1B1,A2B1	.81818
3	A1,A2,A1B1,A2B1	.57576
4	A1,A2,B1	.45921

Table 6

Conversion to ANOVA Lambda's

Source	Models	Calculating Lambda's	Lambda
A'	1/2	.42657/.81818	.52136
B	1/3	.42657/.57576	.74088
A'B	1/4	.42657/.45921	.92892

Table 7

Conversion of Lambda's to ANOVA F's

Source	[(1 - Lambda) / Lambda]	*	(df error / df effect)	= Fcalc
A'	[(1 - .52136) / .52135]	*	(6 / 2)	=
	.91806	*	3	= 2.754
B	[(1 - .74088) / .74088]	*	(6 / 1)	=
	.34975	*	6	= 2.098
A'B	[(1 - .92892) / .92892]	*	(6 / 2)	=
	.07652	*	3	= .230

Table 8

CCA Subsumes Multiple Correlation (R)

[Y with X, A, B]

Canonical Analysis		Regression Analysis	
Squared Rc	.69920	Squared R	.69920
Rc	.83618	R	.83618
Lambda	.30080		
chi-square	10.21116	F	6.19861
df	3	df	3,8
p	.017	p*	.05>p>.01

* Derived from F tables since not provided by printout.

Table 9

Function Coefficient and Beta Weight Conversions

Predictor	Function Coefficient	* Rc (or R)	= Beta Weight	/ Rc (or R)	= Function Coefficient
X	-1.18685	* .83618	= -.99242	/ .83618	= -1.18685
A	1.54635	* .83618	= 1.29303	/ .83618	= 1.54635
B	.14582	* .83618	= .12193	/ .83618	= .14582

Table 10

CCA Subsumes Discriminant Analysis

[B1 with Y, X]

Squared Rc	.29425		
Rc	.54245	Rc	.5424508
Lambda	.70575		
chi-square	3.13648	chi-square	3.1365
df	2	df	2
p	.208	p	.2084
Func. Coefs.		Func. Coefs.	
for Y	-.70221	for Y	-.77101
for X	.70221	for X	.77101

Table 11

Canonical and Discriminant Variate Scores

Case	Canonical Variate Score	Discriminant Variate Score
1	1.94759	2.21043
2	0.58428	0.66313
3	-0.19476	-0.22104
4	0.77904	0.88417
5	-0.19476	-0.22104
6	1.16855	1.32626
7	0.00000	0.00000
8	-1.36331	-1.54730
9	0.00000	0.00000
10	-1.36331	-1.54730
11	-0.97379	-1.10521
12	-0.38952	-0.44209
Mean	0.0000	0.0000
SD	0.0000	1.1350

Note. Canonical variate scores have been "reflected" by multiplication by negative one.

Table 12

Factorial MANOVA

[Y, X with A´(1,3), B(0,1)]

Source	Lambda	Fcalc	df	p
A´	.03202	11.47102	4,10	.001
B	.60902	1.60494	2,5	.289
A´B	.37812	1.56561	4,10	.257

Table 13

Canonical Analyses Using Four Models

Model	Predictors of Y, X	Lambda
1	A1,A2,B1,A1B1,A2B1	.02113
2	B1,A1B1,A2B1	.65989
3	A1,A2,A1B1,A2B2	.03469
4	A1,A2,B1	.05588

Table 14

Conversion to MANOVA Lambda´s

Source	Models	Calculating Lambda´s	Lambda
A´	1/2	.02113/.65989	.03202
B	1/3	.02113/.03469	.60911
A´B	1/4	.02113/.05588	.03781